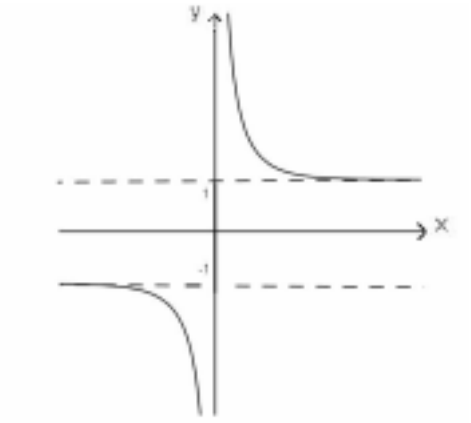
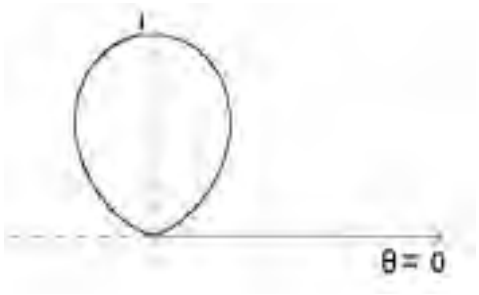
4726 Further Pure Mathematics 2

1(i)	Attempt area = ±Σ(0.3y) for at least three y values	M1	May be implied
	Get 1.313(1..) or 1.314	A1	Or greater accuracy
(ii)	Attempt ± sum of areas (4 or 5 values)	M1	May be implied
	Get 0.518(4..)	A1	Or greater accuracy **SC** If answers only seen, 1.313(1..) or 1.314 B2 0.518(4..) B2 −1.313(1..) or −1.314 B1 −0.518(4..) B1
	Or		
	Attempt answer to part **(i)**−final rectangle	M1 A1	
	Get 0.518(4..)		
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x	M1	Must be quadratic; = 0 may be implied
	Get $x^2(y\text{-}1) - x(2y+1) + (y\text{-}1)=0$	A1	
	Use $b^2 \geq 4ac$ for real x on their quadratic	M1	Allow =,>,<,≤ here; may be implied
	Clearly solve to AG	A1	If other (in)equalities used, the step to AG must be clear **SC** Reasonable attempt to diff. using prod/quot rule M1 Solve correct dy/dx=0 to get $x = -1$, $y = ¼$ A1 Attempt to justify inequality e.g. graph or to show $\mathrm{d}^2y/\mathrm{d}x^2>0$ M1 Clearly solve to AG A1
3(i)	Reasonable attempt at chain rule	M1	Product in answer
	Reasonable attempt at product/quotient rule	M1	Sum of two parts
	Correctly get f′(0) =1	A1	
	Correctly get f″(0) = 1	A1	**SC** Use of ln$y$ = sin$x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values	M1	In $af(0) + bf'(0)x + cf''(0)x^2$
	Get $1 + x + ½x^2$	A1√	From their f(0), f′(0), f″(0) in a correct Maclaurin; all non-zero terms
4	Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow A=1 and/or B=1 quoted
	Get $x^3 = A(x\text{-}2)(x^2+4)+B(x^2+4)+(Cx+D)(x\text{-}2)$	M1	Allow √ mark from their Part Fract; allow D=0 but not C=0
	State/derive/quote A=1	A1	
	Use x values and/or equate coeff	M1	To potentially get all their constants

	Get B=1, C=1, D=-2	A1	For one other correct from cwo
		A1	For all correct from cwo
5(i)	Derive/quote $d\theta=2dt/(1+t^2)$	B1	May be implied
	Replace their cos θ and their $d\theta$, both in terms of t	M1	Not $d\theta = dt$
	Clearly get $\int(1-t^2)/(1+t^2)\ dt$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
			SC
			Derive $d\theta = 2\cos^2\frac{1}{2}\theta\ dt$ B1
			Replace $\cos\theta$ in terms of half-angles and their $d\theta$ ($\neq dt$) M1
			Get $\int 2\cos^2\frac{1}{2}\theta - 1\ dt$ or $\int 1 - 1/2\cos^2\frac{1}{2}\theta\ .2/(1+t^2)\ dt$ A1
			Use $\sec^2\frac{1}{2}\theta = 1+t^2$ M1
			Clearly get/derive AG A1
(ii)	Integrate to $a\tan^{-1}bt - t$	M1	
	Get $\frac{1}{2}\pi - 1$	A1	
6	Get $k\sinh^{-1}k_1x$	M1	For either integral; allow attempt at ln version here
	Get $\frac{1}{3}\sinh^{-1}\frac{3}{4}x$	A1	Or ln version
	Get $\frac{1}{2}\sinh^{-1}\frac{2}{3}x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{\frac{1}{3}}.3^{\frac{1}{2}}$	A1	Or equivalent

7(i)		B1	*y*-axis asymptote; equation may be implied if clear
		B1	Shape
		B1	$y = \pm 1$ asymptotes; may be implied if seen as on graph
(ii)	Reasonable attempt at product rule, giving two terms	M1	
	Use correct Newton-Raphson at least once with their $f'(x)$ to produce an x_2	M1	May be implied
	Get $x_2 = 2.0651$	A1√	One correct at any stage if reasonable
	Get $x_3 = 2.0653$, $x_4 = 2.0653$	A1	cao; or greater accuracy which rounds
(iii)	Clearly derive $\coth x = \frac{1}{2}x$	B1	AG; allow derivation from AG Two roots only
	Attempt to find second root e.g. symmetry	M1	
	Get ± 2.0653	A1√	± their iteration in part **(ii)**
8(i)	**(a)** Get $\frac{1}{2}(e^{\ln a} + e^{-\ln a})$	M1	
	Use $e^{\ln a} = a$ and $e^{-\ln a} = {}^1/_a$	M1	
	Clearly derive AG	A1	
	(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh	M1	4 terms in each
	Correct expansion seen as $e^{(x+y)}$ etc.	A1	
	Clearly tidy to AG	A1	With $e^{-(x-y)}$ seen or implied
(ii)	Use $x = y$ and $\cosh 0 = 1$ to get AG	B1	
(iii)	Attempt to expand and equate coefficients	M1	$(13 = R\cosh\ln a = R(a^2+1)/2a$ $5 = R\sinh\ln a = R(a^2-1)/2a)$
	Attempt to eliminate R (or a) to set up a solvable equation in a (or R)	M1	**SC** If exponential definitions used, $8e^x + 18e^{-x} = Re^x/a + Rae^{-x}$ and same scheme follows
	Get $a = {}^3/_2$ (or $R = 12$)	A1	
	Replace for a (or R) in relevant equation to set up solvable equation in R (or a)	M1	
	Get $R=12$ (or $a = {}^3/_2$)	A1	Ignore if $a={}^2/_3$ also given
(iv)	Quote/derive $(\ln{}^3/_2, 12)$	B1√ B1√	On their R and a
9(i)	Use $\sin\theta.\sin^{n-1}\theta$ and parts	M1	Reasonable attempt with 2 parts, one yet to be integrated

	Get $-\cos\theta.\sin^{n-1}\theta+(n\text{-}1)\int\sin^{n-2}\theta.\cos^2\theta\ d\theta$	A1	Signs need to be carefully considered
	Replace $\cos^2 = 1 - \sin^2$	M1	
	Clearly use limits and get AG	A1	
(ii)	**(a)** Solve for r=0 for at least one θ	M1	θ need not be correct
	Get (θ) = 0 and π	A1	Ignore extra answers out of range
		B1	General shape (symmetry stated or approximately seen)
		B1	Tangents at θ=0, π and max r seen
	(b) Correct formula used; correct r	M1	May be $\int r^2\ d\theta$ with correct limits
	Use $6I_6 = 5I_4$, $4I_4 = 3I_2$	M1	At least one
	Attempt I_0 (or I_2)	M1	(I_0 = ½π)
	Replace their values to get I_6	M1	
	Get 5π/32	A1	
	Use symmetry to get 5π/32	A1	May be implied but correct use of limits must be given somewhere in answer
	Or		
	Correct formula used; correct r	M1	
	Reasonable attempt at formula $(2i\sin\theta)^6 = (z - {}^1/z)^6$	M1	
	Attempt to multiply out both sides (7 terms)	M1	
	Get correct expansion	A1	
	Convert to trig. equivalent and integrate their expression	M1	cwo
	Get 5π/32	A1	
	Or		
	Correct formula used; correct r	M1	
	Use double-angle formula and attempt to cube (4 terms)	M1	
	Get correct expression	A1	
	Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
	Reasonable attempt to integrate $\cos^3 2\theta$ as e.g.$\cos^2 2\theta.\cos 2\theta$	M1	cwo
	Get 5π/32	A1	